

## Document extract

Title of chapter/article	What is 'typical' for different kinds of data?
Author(s)	Jane M. Watson
Copyright owner	Education Services Australia
Published in	Top Drawer Teachers <a href="http://topdrawer.aamt.edu.au">http://topdrawer.aamt.edu.au</a>
Year of publication	2013
Page range	
ISBN/ISSN	

This document is protected by copyright and is reproduced in this format with permission of the copyright owner(s). This document may be used, reproduced, published, communicated and adapted free of charge for non-commercial educational purposes provided all acknowledgements associated with the material are retained.



---

### **AAMT—supporting and enhancing the work of teachers**

---

The Australian Association of Mathematics Teachers Inc.

ABN 76 515 756 909

POST GPO Box 1729, Adelaide SA 5001

PHONE 08 8363 0288

FAX 08 8362 9288

EMAIL [office@aamt.edu.au](mailto:office@aamt.edu.au)

INTERNET [www.aamt.edu.au](http://www.aamt.edu.au)



# What is ‘typical’ for different kinds of data? Examples from the Melbourne Cup

Jane Watson, University of Tasmania

There are five words that are critical to an appreciation of what is ‘typical’ in the data sets that students encounter across the middle school years. The first three words – mean, median and mode – are nouns that define measures of typicality in data sets. The next two words – categorical and numerical – are adjectives that describe the types of data sets to which we may wish to apply the measures.

The purpose of this article is to present and discuss in a down-to-earth fashion, with authentic data sets, the two types of data and the relationship between the three measures of typicality introduced in the curriculum. Although the mean, median and mode have been in mathematics text books for many years, it is sobering to read in the research literature that teachers’ understanding of the measures is little different from that of students, and that both struggle (Jacobbe & Carvalho, 2011).

## What does the Australian Curriculum say?

Mean, median, and mode are not explicitly mentioned in the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2013) until Year 7, where we find the following:

- Calculate mean, median, mode and range for sets of data. Interpret these statistics in the context of data (ACMSP171)
- Describe and interpret data displays using median, mean and range (ACMSP172)

The inclusion of ‘context of data’ assumes an appreciation of which types of measures are associated with which contexts. Looking back through the curriculum we find that categorical data or variables are mentioned in Year 2, Year 3 and Year 5, whereas numerical data or variables are first mentioned in Year 5. This progression must reflect the writers’ appreciation that counting frequencies and comparing them across categories (e.g., favourite fruit) is achievable in these early years. It would have been useful, however, to introduce the term ‘mode’ when giving the example of ‘most popular breakfast cereal’ in Year 3, or when expecting students to interpret data displays (ACMSP070). The mode as ‘most popular’ or ‘most frequent’ is not a complex idea and could be seen as a way of describing what is typical for categorical data sets.

By Year 7, where mean, median and mode are introduced, numerical data are the focus, and although the mode is mentioned when students are asked to calculate the statistics and interpret in the context of data, it is not mentioned when students are asked to describe and interpret data displays. Once numerical data become the focus of statistical investigations, with the potential to employ the mean and median, it may not be surprising that the interest in the mode lessens. The emphasis on categorical data throughout the primary years,

however, suggests that there may be confusion on the part of students about whether they can apply all of these new measures to the data sets with which they are most familiar.

## How do we know which measure to use for which type of data?

The examples provided here are based on data collected for winners of the Melbourne Cup (Watson, Beswick, Brown, Callingham, Muir, & Wright, 2011). In this Australian context, variables are available that are both categorical and numerical. If we think about the curriculum and start with categorical data, the measure of typical we need is the **mode**, which tells us which category or categories has/have the most values of the variable in it/them. In Figure 1 are five different sexes for the winners of the Melbourne Cup: Filly, Mare, Colt, Gelding, and Stallion.<sup>1</sup> We see from the column graph that the most common sex of winners of the Melbourne Cup is Stallion. We could say this is the modal or typical sex of winners.

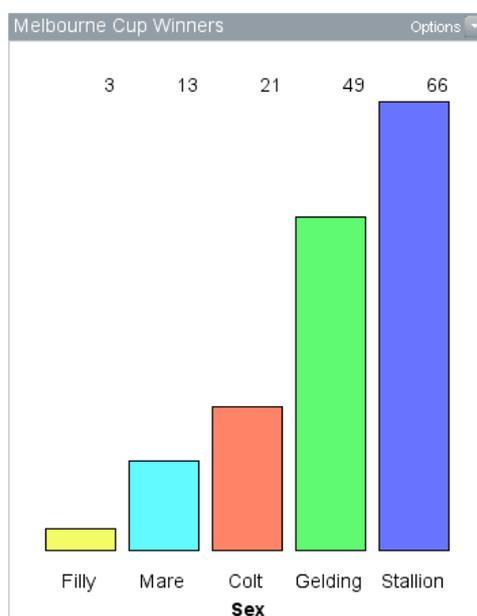


Figure 1. Sex of Melbourne Cup winners.

Other sets made up of numerical rather than categorical values can also have modes. For example, in looking at the Age of winners of the Melbourne Cup in Figure 2, we see two modes. This makes it a bit more difficult to talk about the typical age, although we could say it is “4 or 5 years.”

---

<sup>1</sup> Colt: a male horse under the age of four. Filly: a female horse under the age of four. Mare: a female horse four years old and older. Stallion: a non-castrated male horse four years old and older. Gelding: a castrated male horse of any age.

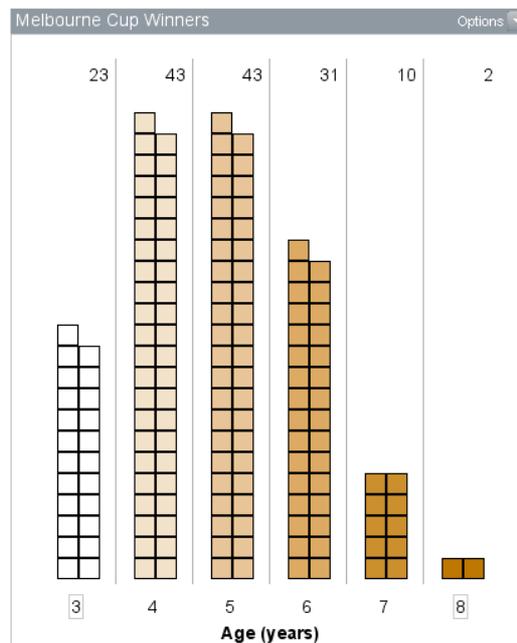


Figure 2. Age of Melbourne Cup winners.

There are also other ways to talk about typical values in data sets. For example, we could talk about putting numerical data in order from the smallest to the largest and finding the value in the middle. This value is called the **median**. It would give us a feeling for what is a typical value in the middle of the set. If we looked for the median of the ages of the winners of the Melbourne Cup, with 152 values, we would want the value with 76 ages on either side. Looking at the plot in Figure 2, we see that there are a total of 66 values in the bins for 3-year-old and 4-year-old horses. That means that the 76<sup>th</sup> and 77<sup>th</sup> values are in the 5-year-old bin. So the middle value in the data set, the median, is 5 years.

Returning to Figure 1, is it possible to talk about the median sex of the winners of the Melbourne Cup? One might claim that Colt is the median because it is in the middle. The 76<sup>th</sup> and 77<sup>th</sup> values from the left in the plot, however, lie in the block for Geldings. Being the centre category in a bar chart does not make a value the median. Suppose the Sex categories were replotted alphabetically as in Figure 3. Then the 76<sup>th</sup> and 77<sup>th</sup> values would lie in the block for Mares. What would this tell us? Only that it does not make any sense to talk about the median of a categorical data set. Since the data cannot be ordered numerically, there can be no middle value. Leavy, Friel, and Mamer (2009) give other examples of students trying to find the median of categorical data sets.

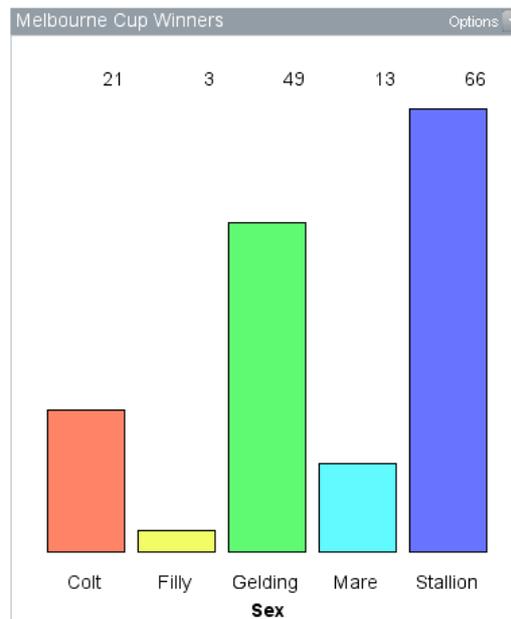


Figure 3. Sex of Melbourne Cup winners (ordered alphabetically).

As numerical data become more detailed in their representation, and clustered or spread out, the most complex way of considering what is typical is likely to be the calculation of the **mean**. The mean is the total of all of the numerical values in the data set divided by the number of numerical values. Physically, this would be the balance point if all of the values could be placed on a scaled ruler with a fulcrum at this numerical value, the mean. Some activities based on the mean as balance point are found in O'Dell (2012) and Hudson (2012).

The mean Age of the winners of the Melbourne Cup turns out to be 4.79 (728 divided by 152). Notice that this number, shown in Figure 4, is not one of the ages of the winners. This might be considered annoying if we wanted to use the mean to talk about the typical age of a winner from the data given, which are in whole numbers. So perhaps the best way to talk about the typical age of a winner of the Melbourne Cup would be to use the median, if we wanted a single value that would be the most intuitive to the general people interested in the Cup.

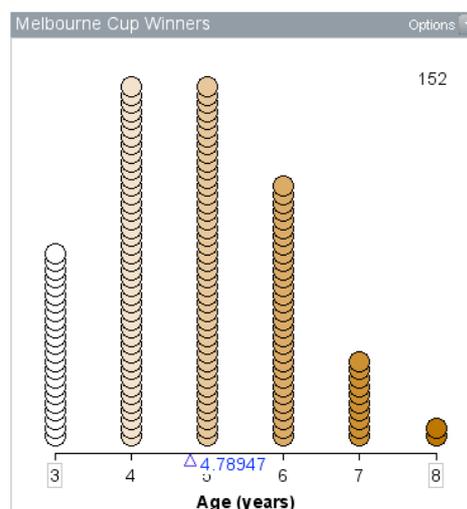


Figure 4. Ages of Melbourne Cup winners with the mean marked on the axis.

When data sets become more detailed numerically, often the mean is the best way to describe the data in one number because it takes into account every number in the data set. Consider the plot in Figure 5 of the Weight carried by the winners of the Melbourne Cup. It is quite crowded in the middle and we see that the mean is marked as 51.2 kg. It is pretty easy to imagine this number as a balance point because the values are fairly evenly distributed on either side of it. It also might seem reasonable in this case that 51.2 is one of the values in the data set.

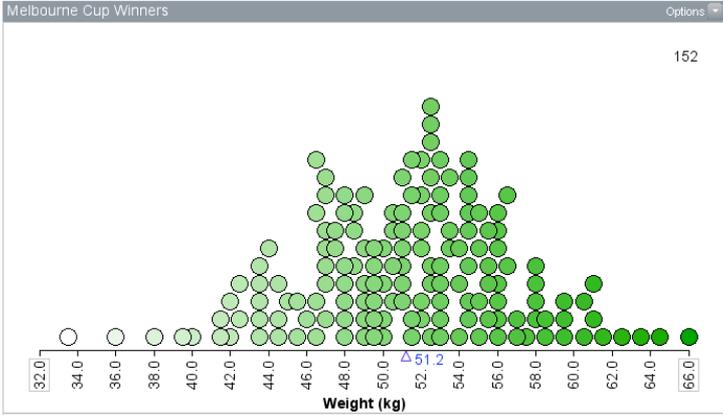


Figure 5. Weight carried by the Melbourne Cup winners.

Would the mode be useful to describe the typicality of this data set? We would need to separate the data better to tell. Looking at the plot in Figure 6 we see that there are three modes (52.5, 53.0 and 54.5). These values, however, have only one greater frequency than four other values (47.0, 48.0, 51.0 and 56.0). The mode is hence not very useful in telling us about the typicality of the Weights carried by the winners of the Melbourne Cup, except that it does tell us that there are quite a few values that are repeated about the same number of times in the centre of the data set.

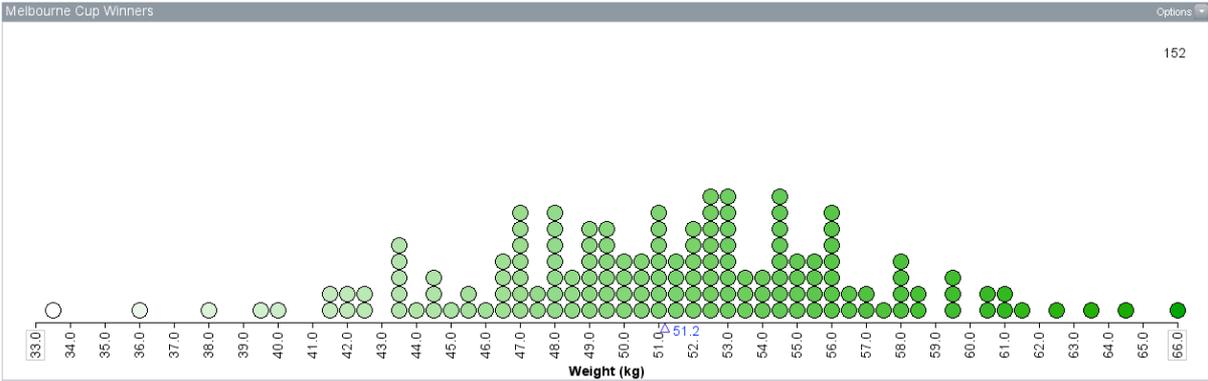


Figure 6. Weight carried by Melbourne Cup winners showing three modes.

What would the median tell about the typical value/s in this data set? As we can see in the plot in Figure 7, the middle value is one of the 51.5 values, which is very close to the mean value of 51.2 kg. Now we see that again (as for Age) the mean is not one of the values in the set.

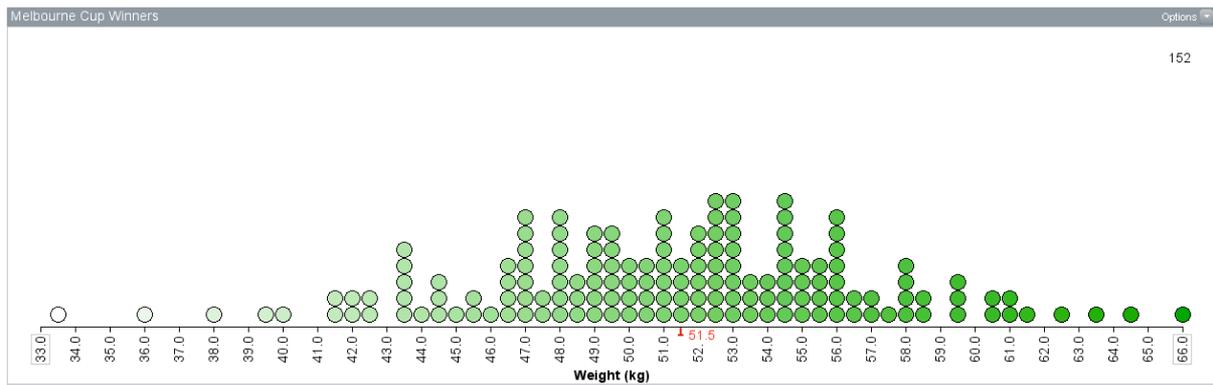


Figure 7. Weight carried by Melbourne Cup winners showing the median of 51.5 kg.

The closeness of the mean and the median tells us that the distribution of the Weights is relatively even on both sides of these middle points. In other words, the distribution is fairly symmetric. However, looking at the plot for the Winning Margin (measured in ‘lengths’), we see a different shape. Because there have been many close finishes to the Melbourne Cup, in Figure 8 there are many values close to zero.

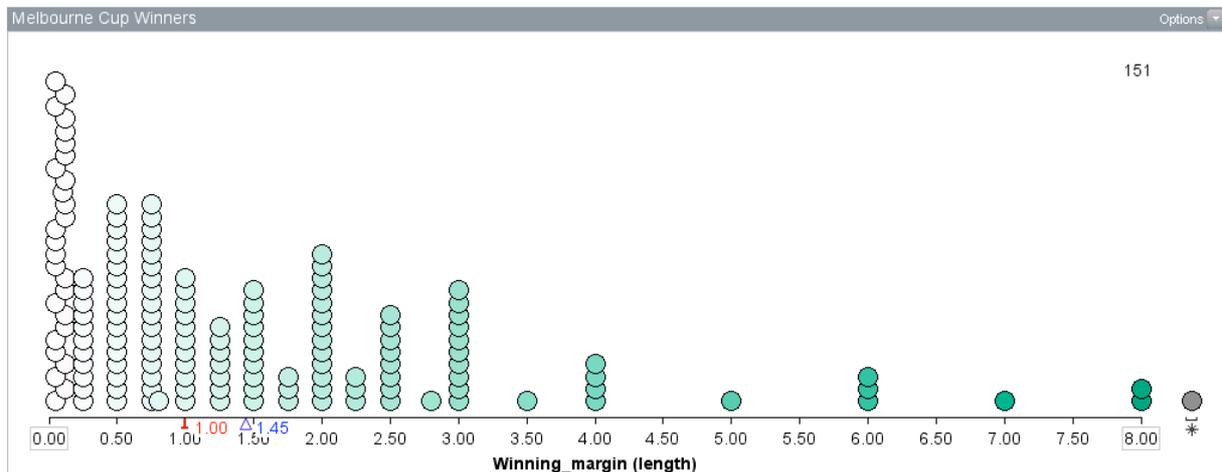


Figure 8. Winning margin, measured in lengths for the Melbourne Cup.

In the Winning Margin plot, we see that the median is 1.00 and the mean is 1.45. This is because the median takes into account the order of the data but not their exact values. Here we see that the median is the 76<sup>th</sup> value (there is one missing value), which is one of a number of 1.00 values. The mean is larger because it takes into account every value in the data set and is influenced by the large values on the right. The median is not influenced by the actual values, only that there are values to the left or right of the 76<sup>th</sup> value. The mode is of little use here because of the many values very close to zero. All we can say with reference to this is that it is typical for races to have a close finish! This comes from looking at the entire distribution.

Returning to Figure 1, showing the Sex of winners of the Melbourne Cup, we realise that it would be impossible to try and calculate a mean. One can't multiply Filly x 3, Mares x 13, etc., add them together, and then divide by 152. The mean is not used at all for categorical data sets. Nor is the median.

More likely the categorical variables are used to separate the data set in ways that are of interest when exploring the numerical variables. For example, consider the plot in Figure 9 where the Weight carried by the winner is on the horizontal axis and the Sex of the winner is on the vertical axis, creating 5 sub-groups of data. In this case we notice that we can order the Sex of winners to show an increasing mean value for Weight carried from Filly, to Colt, to Mare, to Gelding, to Stallion. An expert could probably explain why this might be so! Categorical variables have a very important part to play in statistics but that importance is not related to being able to produce a mean or median value as a representative statistic.

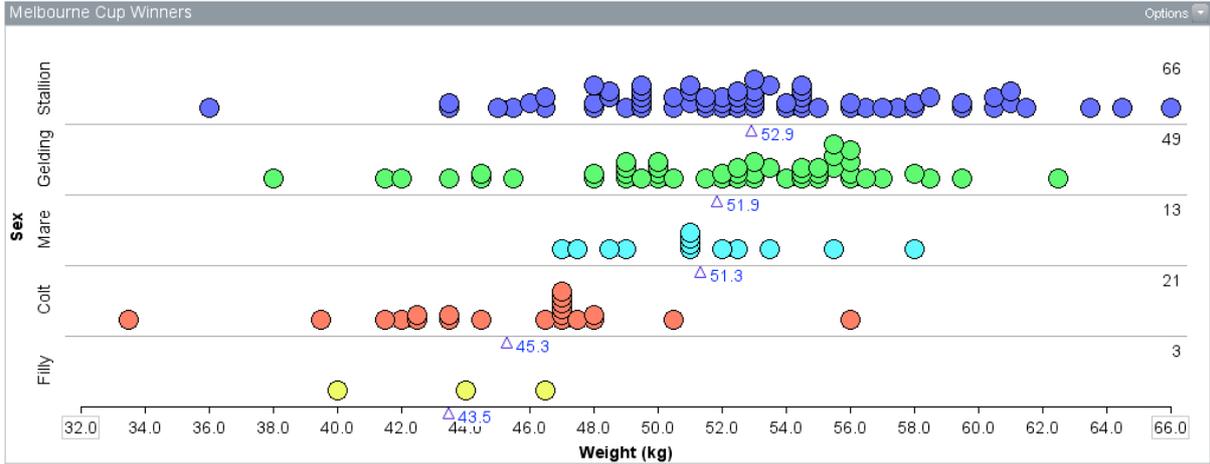


Figure 9. Weight carried by Melbourne Cup winners separated by Sex.

Some numbers are used as values for a variable but are treated as categorical rather than numerical. Consider for example the variable Year – the year that the Melbourne Cup was run – containing the values from 1861 to 2012. We could work out the median year or calculate the average year by adding the years and dividing by 152. In this case, the median would be half way between 1936 and 1937, or 1936.5, which is meaningless in the context of the Melbourne Cup. Because the data are totally symmetric the mean will have the same value of 1936.5. Further, the mode for the variable Year is similarly useless because every year in the data set has a frequency of one! It tells us nothing about typicality.

A categorical variable like Year is useful to us precisely because it is uniform along a scale and it helps us keep track of other variables in a ‘time series’. For example, if we look at the Weights carried by the winners of the Melbourne Cup, we see what seems to be a reduction in the variation in weights over the years. Hence, although Year is not a variable of much interest on its own (except to keep track of how many years the Cup has been run), it is useful in other ways.

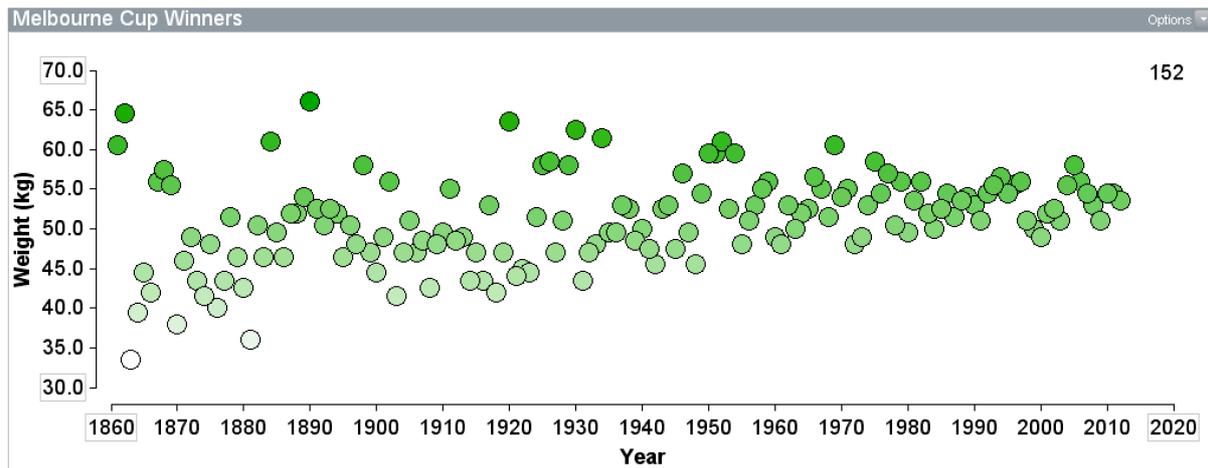


Figure 10. Plot of Weight carried each year by the Melbourne Cup winner.

It is interesting to note the increasing complexity of the procedures associated with the concepts of mode, median and mean, in terms of their use of the information contained in a data set, although each potentially produces only one number.

- The mode only looks for the ‘most’ frequent values, so could be identified visually from an appropriate plot in one step.
- The median looks for ‘middle’ and uses all of the values in the data set but only their order. Only counting is needed once the values are placed in order. There are therefore two steps: place the values in order and then count to find the middle.
- The mean looks for ‘balance’ and uses all of the values in the data set including their numerical values. They need to be added carefully and then divided by the number of values. Even with a calculator this can be a tedious job. There are three steps: record the values, add the values, divide by the number of values.

## A wider view of ‘typical’

For numerical data we can also take a somewhat broader view of typical by considering the ‘middle’ values of data, rather than just looking at a single value. The box plot uses the median as its centre because it is the value half-way through the ordered data. A rectangular box is then created to represent the quarter of data on either side of the median. The box hence ‘covers’ the middle half of the data, which gives us a general appreciation of the typical values in the centre of the distribution. At either end of the box, a whisker is drawn to the extreme values at the ends of the data set, marking the lowest quarter and the highest quarter of the data. This gives us a feel for the overall variation in the data when we are focussing on typical values.

Box plots vary greatly depending on the shape of a data distribution. This is seen in Figure 11 which shows the box plots for the Weight carried by winners of the Melbourne Cup, and the Winning Margin in the race. On one hand, the box representing the middle half of the data is in the middle of the plot for Weight, and the median is in the middle of the box, showing us that the data for this variable are quite symmetric (check this in Figure 5). On the other hand, the middle half of the data for Winning Margin is nowhere near the middle of the plot, indicating a data set that is skewed away from zero. The plot shows the lower 25% of the data

squeezed up near zero and the upper 25% of the data very spread out from 2.0 to 8.0 lengths. More description of box plots is found in Watson (2012).



Figure 11. Box plots for Weight carried and Winning Margin for Melbourne Cup winners.

Box plots are useful for a quick summary comparing the shape of data when separated by a categorical variable. Figure 12 shows the box plots for Weight carried for each of the categories of the Sex variable. The impression is slightly different from the representation in Figure 9 because the middle of the box is the median in Figure 12, not the mean as in Figure 9. Would this make us change our minds about whether Stallions carried more weight than Geldings when winning the Melbourne Cup? The variation in the data is greater for Stallions than Geldings and we see the difference in the impression of typical when the mean is used rather than the median.

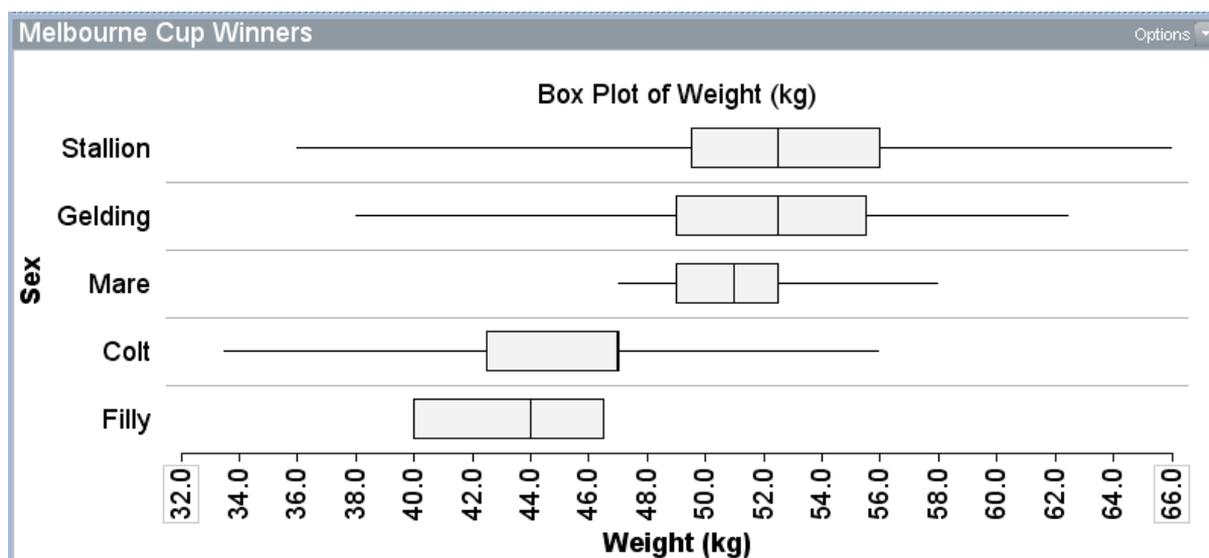


Figure 12. Box plots for Weight carried by Melbourne Cup winners separated by Sex.

Starting with the idea of ‘typical’ as the mode, we have progressed through considering the median and mean, to expanding ‘typical’ to include the middle half of a numerical data set. Although the box plot is not introduced in *The Australian Curriculum: Mathematics* (ACARA, 2013) until Year 10, students should become cognizant of the importance of the overall shape of a data set when considering typicality. What is typical for the Weight carried by Melbourne Cup winners is different from what is typical for their Winning margins. The Weight is typically symmetric whereas the Winning Margin is typically skewed.

## Acknowledgement

The plots in this article were created by the software *TinkerPlots* (Konold & Miller, 2011).

## References

- Australian Curriculum, Assessment and Reporting Authority (ACARA). (2013). *The Australian curriculum: Mathematics, Version 4.1, 1 February 2013*. Sydney, NSW: ACARA.
- Hudson, R. A. (2012). Finding the balance at the elusive mean. *Mathematics Teaching in the Middle School, 18*, 301–306.
- Jacobbe, T. & Fernandes de Carvalho, C. (2011). Teachers understanding of average. In C. Batanero, G. Burrill, C. Reading & A. Rossman (Eds), *Teaching statistics in school mathematics: Challenges for teaching and teacher education* (pp. 199–209). New York: Springer.
- Konold, C. & Miller, C. D. (2011). *TinkerPlots: Dynamic data exploration* [computer software, Version 2.0]. Emeryville, CA: Key Curriculum Press.
- Leavy, A. M., Friel, S. M. & Mamer, J. D. (2009). It’s a Fird! Can you compute a median of categorical data? *Mathematics Teaching in the Middle School, 14*, 244–351.
- O’Dell, R. S. (2012). The mean as balance. *Mathematics Teaching in the Middle School, 18*, 148–155.
- Watson, J. (2012). Box plots in the Australian curriculum. *The Australian Mathematics Teacher, 63*(3), 3–11.
- Watson, J., Beswick, K., Brown, N., Callingham, R., Muir, T. & Wright, S. (2011). *Digging into Australian data with TinkerPlots*. Melbourne: Objective Learning Materials.