Proving congruence: Answers

http://topdrawer.aamt.edu.au/Geometric-reasoning/Misunderstandings/Similar-or-congruent/Complete-the-congruence-proof

Complete these proofs, putting in the reasons and missing angles.
Mark the equal angles and sides you find on the diagram as you go.

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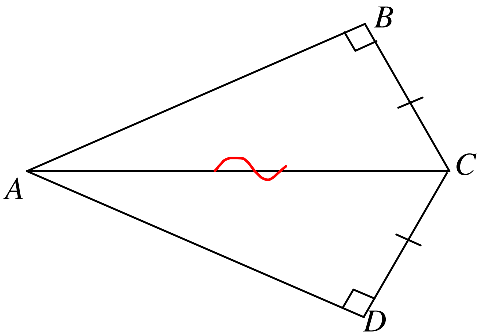
1. Given: $BC = DC$; $AB \perp BC$ and $CD \perp DA$
   Aim: To prove $\Delta ABC \equiv \Delta ADC$



Proof:
In ΔABC and ΔADC
1. $\angle B = \angle D = 90°$
   (given)
2. AC is common
   (given)
3. $BC = DC$

∴ $\Delta ABC \equiv \Delta ADC$ (R H S)

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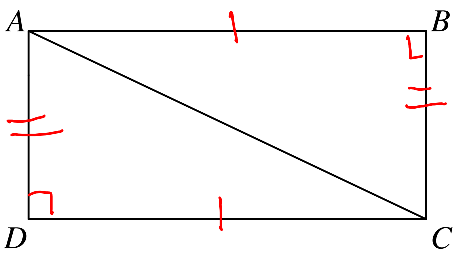
2. Given: ABCD is a rectangle
   Aim: To prove $\Delta ABC \equiv \Delta ADC$



Proof:
In ΔABC and ΔADC
1. $AB = DC$
   (opposite sides rectangle)
2. $AD = BC$
   (as above)
3. AC

∴ $\Delta ABC \equiv \Delta ADC$ (S S S)

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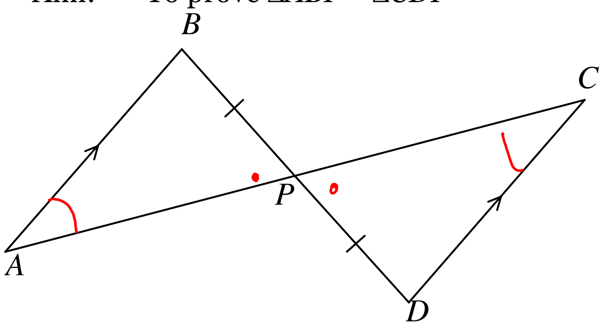
3. Given: $AB \,||\, DC$ and $BP = PD$
   Aim: To prove $\Delta ABP \equiv \Delta CDP$



Proof:
In ΔABP and ΔCDP
1. $\angle A = \angle C$
   (alternate angles; $AB \,||\, CD$)
2. $\angle APB = \angle DPC$
   (vertically opposite)
3. $BP = PD$
   (given)

∴ $\Delta ABP \equiv \Delta CDP$ (A A S)

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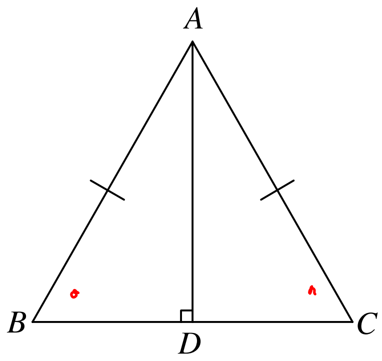
4. Aim: To prove $BD = DC$



Proof:

In ΔABD and ΔACD

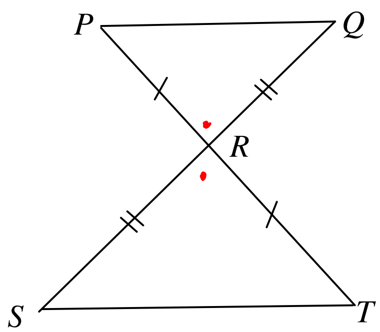
1. $\angle ADC = \angle ADB = 90°$
   (given)
2. $AB = AC$
   (given)
3. $AD = AD$
   (common)

$\therefore$ $\Delta ABD \equiv \Delta ACD$ (<u>R</u> <u>H</u> <u>S</u>)

$\therefore$ $BD = CD$
(matching sides of congruent Ds)

You cannot use the property of *AD* bisecting *BC*, as this is the goal of the question!

5. Aim: To prove $PQ \parallel ST$



Proof:

In ΔPQR and ΔSTR

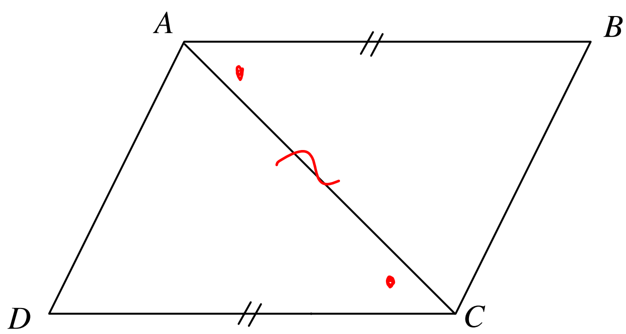
1. PR = RT
   (given)
2. $\angle PRQ = \angle SRT$
   (vertically opposite)
   $QR = RS$
   (given)

$\therefore$ $\Delta PQR \equiv \Delta TSR$ (<u>S</u> <u>A</u> <u>S</u>)

$\therefore$ $\angle PQR = \angle RST$
(matching angles of congruent Ds)
But these are <u>alternate</u> angles

$\therefore$ $PQ \parallel ST$
(alternate angles are equal)

6. Aim: To prove $\angle B = \angle D$



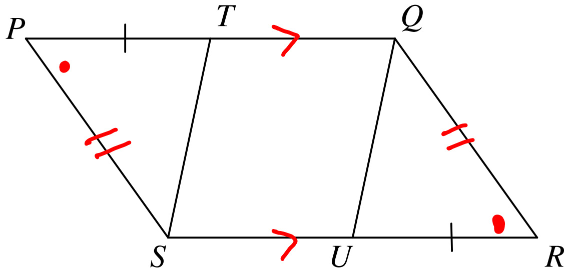
In ΔABC and ΔADC

1. $DC = AB$ (given)
2. $\angle BAC = \angle ACD$ (alternate angles; $AB \parallel CD$)
3. AC is common

$\therefore$ $\Delta ABC \equiv \Delta ADC$ (<u>S</u> <u>A</u> <u>S</u>)

$\therefore$ $\angle B = \angle D$ (matching angles of congruent Ds)

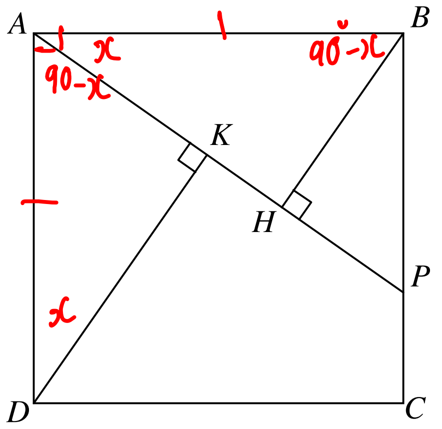
7. Given: *PQRS* is a parallelogram. *PT* = *RU*.
Aim: To prove *TS* = *QU*



In Δ*PTS* and Δ*RUQ*
1. *PT* = *UR* (given)
2. ∠*TPS* = ∠*QRS*
   (opp. angles parallelogram)
3. PS = QR
   (opp. sides parallelogram)
   Δ*PTS* ≡ Δ*RUQ* (S A S)

∴ *TS* = *QU* (matching sides of cong. triangles)

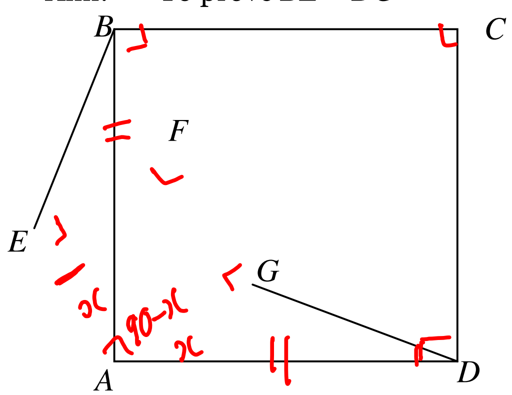
8. Given: *ABCD* is a square.
BH ⊥ *AP* and *DK* ⊥ *AP*.
Aim: To prove *AH* = *DK*



Proof:
In Δ*ABH* and Δ*ADK*
1. ∠*AHB* = ∠*AKD* = 90°
   (given)
2. ∠*HAB* + ∠*ABH* + ∠*AHB* = 180°
   (angle sum D *AHB*)
   ∴ ∠*ABH* = 90° – ∠*HAB*
   But ∠*DAK* = 90° – ∠*HAB*
   (∠*DAB* = 90°, *ABCD* is a square)
   ∴ ∠*ABH* = ∠*DAK*
3. *AB* = *AD*
   (sides of a square)
   ∴ Δ*ABH* ≡ Δ*DAK* (A A S)

∴ *AH* = *DK*
(matching sides of cong. triangles)

9. Given: *ABCD* and *AEFG* are both squares.
Aim: To prove *BE* = *DG*



Let *x* = ∠*EAB*
∴ ∠*BAG* = 90° – *x*
(∠*EAG* = 90°, square *EAGF*)
∴ ∠*GAD* = 90° – (90° – *x*) = *x*
(∠*BAD* = 90°, square *ABCD*)

In Δ*AEB* and Δ*AGD*
1. *AB* = *AD* (sides of square ABCD)
2. ∠*EAB* = ∠*GAD* (see above)
3. *AE* = *AG* (sides of square *AEFG*)

Δ*AEB* ≡ Δ*AGD* (S A S)
∴ *BE* = *DG* (matching sides of cong. triangles)